LOGIC IN INDIAN THOUGHT

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This paper presents an overview of the Indian tradition of logic. The paper starts with the Vedic ideas related to logic and goes on to summarize the relevant contributions of the formal schools of philosophy which included one devoted principally to logic. The Indian tradition of logic reached its peak in the Navya Nyāya school of medieval India.

1. Introduction

This article is a general survey of the tradition of logic ($\bar{a}nv\bar{\imath}k\d{s}ik\bar{\imath},\ ny\bar{a}ya$, or $tarka$ in Sanskrit) in India. This tradition is very old and can be seen in its beginnings in the *Ṛgveda*, the earliest text available from India and dated to about 2000 BCE. The Vedic system looked at reality at two levels. At the ordinary level of apprehension it was rational and, therefore, it needed logic to describe it; but at a higher level it had a transcendental basis. The transcendental nature was expressed in statements that were paradoxical such as the individual self was equivalent to the cosmic self (*ātman* equals *brahman*) or fullness is present everywhere, it arises from itself, and when subtracted from itself it remains full (*Īśāvāsya Upaniṣad*). Some have taken this latter statement to imply an intuition of the idea of infinity.

The *Ṛgvedic hymn* 10.129, speaking of creation, mentions a time that was neither existent nor non-existent, suggesting the beginnings of representation in terms of various logical divisions that were later represented formally in Indian logic as the four circles of $catu\d{s}ko\d{t}i$: "A," "not A," "A and not A," and "not A and not not A."

Amongst the early sources of textual evidence for Indian logic are the vari-

ous schools of philosophy including Nyāya and Vaiśeṣika, dealing respectively with linguistic and physical objects. The epic *Mahābhārata* mentions different schools of logic. The grammar of Pāṇini (5th century BCE) uses logical categories and the rich grammatical tradition continued to influence logic and other philosophical thought. Early modern reviews of the subject are by Vidyabhusana [1],[2]; for general reviews, see the edited volumes by Potter [3],[4]; for a broad historical context, see [5],[6],[7].

The tradition of Indian logic, which developed in the background of the Vedic theory of knowledge, was divided by the historian Vidyabhusana [2] into three periods: ancient (up to 400 CE), medieval (400 CE – 1200 CE), and modern (1200 CE – 1850 CE). He saw the *Nyāya Sūtra* of Akṣapāda Gautama (or Gotama) (c 550 B.C.E.) as the foremost, if not the earliest, representative of the ancient period; *Pramāṇa-samuccaya* of Dignāga as representative of the medieval period; and *Tattva-cintāmaṇi* of Gaṅgeśa Upādhyāya as representative of the modern period. The medieval period produced many important glosses on the ancient period and much original thought. For example, Bhartṛhari (5th century CE) presented a resolution to the problem of self-referral and truth (Liar's paradox) [8]. In the modern period philosophers took up new issues such as empty terms, double negation, classification, and essences.

2. Darśanas and the *Nyāya Sūtra*

Logic is one of the six *darśanas*, which are the classical schools of Indian philosophy. These six schools are the different complementary perspectives on reality, which may be visualized as the views from the six walls of a cube within which the subject is enclosed. The base is the system is the broad system of the tradition (Pūrva Mīmāṃsā), and the ceiling represents the large questions of meaning related to the objective world and the subject (Uttara Mīmāṃsā or Vedānta); one side is analysis of linguistic particles (Nyāya), with the opposite side being the analysis of material particles (Vaiśeṣika); another side is enumerative categories in evolution at the cosmic and individual levels (Sāṃkhya), with the opposite side representing the synthesis of the material and cognitive systems in the experiencing individual (Yoga).

Logic is described in Kauṭilya's *Arthaśāstra* (c. 350 BCE) as an independent field of inquiry *ānvīkṣikī* [9]. The epic *Mahābhārata*, which is most likely prior

to 500 BCE because it is not aware of Buddhism in its long descriptions of religion [10], declares (*Mahābhārata* 12.173.45) that $ānvīkṣikī$ is equivalent to the discipline of $tarka$. Clearly, there were several equivalent terms in use in India for logic in 500 BCE.

The canonical text on the Nyāya is the *Nyāya Sūtra* of Akṣapāda Gautama [11]. The most important early commentary on this text is the *Nyāya Bhāṣya* of Vātsyāyana which is estimated to belong to 5th century CE.

The physician Caraka, in his *Saṃhitā*, speaks of the importance of the use of logic in medicine just as it was also essential to other sciences. The *Nyāya Sūtra* speaks of three kinds of debate:

- *kathā* (literally, speech), where a thesis and a counter-thesis are argued by the protagonists based on evidence and argument;
- *jalpa*, which may entail equivocation and false reasoning;
- *vitaṇḍa*, which is characterized by the absence of a counter-thesis.

The Nyāya also calls itself $pramāṇa\ śāstra$, or the science of correct knowledge. Knowing is based on four conditions: (i) The subject or the $pramaṭṛ$; (ii) The object or the $prameya$ to which the process of cognition is directed; (iii) The cognition or the $pramiti$; and (iv) the nature of knowledge, or the $pramāṇa$. The four *pramāṇas* through which correct knowledge is acquired are: $pratyakṣa$ or direct perception, $anumāna$ or inference, $upamāna$ or analogy, and $śabda$ or verbal testimony.

The function of definition in the Nyāya is to state essential nature ($svarūpa$) that distinguishes the object from others. Three fallacies of definition are described: $ativyāpti$, or the definition being too broad as in defining a cow as a horned animal; $avyāpti$, or too narrow; and $asambhava$, or impossible.

Gautama mentions that four factors are involved in direct perception: the senses ($indriyas$), their objects ($artha$), the contact of the senses and the objects ($sannikarṣa$), and the cognition produced by this contact ($jñāna$). The five sense organs, eye, ear, nose, tongue, and skin have the five elements light, ether, earth, water, and air as their field, with corresponding qualities of color, sound, smell, taste and touch.

$Manas$ or mind mediates between the self and the senses. When the *manas* is in contact with one sense-organ, it cannot be so with another. It is therefore said to

be atomic in dimension. It is due to the nature of the mind that our experiences are essentially linear, although quick succession of impressions may give the appearance of simultaneity.
Objects have qualities which do not have existence of their own. The color and class associated with an object are secondary to the substance. According to Gautama, direct perception is inexpressible. Things are not perceived as bearing a name. The conception of an object on hearing a name is not direct perception but verbal cognition.
Not all perceptions are valid. Normal perception is subject to the existence of (i) the object of perception, (ii) the external medium such as light in the case of seeing, (iii) the sense-organ, (iv) the mind, without which the sense-organs cannot come in conjunction with their objects, and (v) the self. If any of these should function improperly, the perception would be erroneous. The causes of illusion may be $doṣa$ (defect in the sense-organ), $samprayoga$ (presentation of only part of an object), or $saṃskāra$ (habit based on irrelevant recollection).
$Anumāna$ (inference) is knowledge from the perceived about the unperceived. The relation between the two may be of three kind: the element to be inferred may be the cause or the effect of the element perceived, or the two may be the joint effects of something else.
The Nyāya syllogism is expressed in five parts:

1. $pratijñā$, or the proposition: the house is on fire;
2. $hetu$, or the reason: the smoke;
3. $dṛṣṭānta$ the example: fire is accompanied by smoke, as in the kitchen;
4. $upanaya$, the application: as in kitchen so for the house;
5. $nigamana$, the conclusion: therefore, the house is on fire.

This may be represented symbolically as [12]:

1. A
2. Because B
3. B goes with A always; witness C
4. It is a case of B

5. Therefore, A

The Nyāya syllogism recognizes that the inference derives from the knowledge of the universal relation ($vyāpti$) and its application to the specific case ($pakṣadharmatā$). There can be no inference unless there is expectation ($ākāṅkshā$) about the hypothesis which is expressed in terms of the proposition.
The minor premise ($pakṣadharmatā$) is a consequence of perception, whereas the major premise ($vyāpti$) results from induction. But the universal proposition cannot be arrived at by reasoning alone. Frequency of the observation increases the probability of the universal, but does not make it certain. Gaṅgeśa, a later logician, suggested that the apprehension of the universal requires $alaukika\ pratyakṣa$ (or nonsensory apprehension).
It was also argued that the major premise (*vyāpti*) should be formulated negatively to ensure that the process of inference does not involve *petitio principii*. Let A be what has a; whatever does not differ from non-A, does not have a. The five-part syllogism would then run as:

1. Not A
2. Because not B
3. A goes with B always; witness C
4. It is not so (not a case of B)
5. Therefore, it is not a case of A

The Nyāya system lays stress on antecedence in its view of causality. But both cause and effect are viewed as passing events. Cause has no meaning apart from change; when analyzed, it leads to a chain that continues without end. Causality is useful within the limits of experience, but it cannot be regarded as of absolute validity. Causality is only a form of experience.
The advancement of knowledge is from $upamāna$, or comparison, with something else already well-known. The leads us back to induction through $alaukika\ pratyakṣa$ as the basis of the understanding.
$Śabda$, or verbal testimony, is a chief source of knowledge. The meaning of words is by convention. The word might mean an individual, a form, or a type, or all three. A sentence, as a collection of words, is cognized from the trace

($saṃskāra$) left at the end of the sentence. Knowledge is divided into cognitions which are not reproductions of former states of consciousness ($anubhava$) and those which are recollections ($smṛti$).
The Nyāya speaks of errors and fallacies arising by interfering with the process of correct reasoning. The Nyāya attacks the Buddhist idea that no knowledge is certain by pointing out that this statement itself contradicts the claim by its certainty. Whether cognitions apply to reality must be checked by determining if they lead to successful action. $Pramā$, or valid knowledge, leads to successful action unlike erroneous knowledge ($viparyāya$).

3. Object and subject

The Nyāya propositions assume a dichotomy between object and subject. The objective world is open to logical analysis since it maps to linguistic categories; the subjective world can suffer from invalid perception for a variety of reasons. This is consistent with the Vedic view that the although the inner world maps the outer, the mind can be clouded by habits or wrong deductions owing to incorrect assumptions.
The Sāṃkhya [13], attributed to the legendary rishi Kapila, is the background to be considered when speaking of Indian logic. Its concern is the enumeration of categories as they arise in the space of the mind with the objective of obtaining discriminative knowledge of the manifest (*vyakta*), the unmanifest (*avyakta*) and the knower (*puruṣa*). In Sāṃkhya, evolution occurs due to changing balance and proportion both in the objective and the subjective worlds. The three $guṇas$ or fundamental modalities are $sattva,\ tamas$ and $rajas$, and they operate both at the large scale as well as in quick transformation. The normative "thing" behind this ceaseless change is the witness, or self, who is viewed in the singular for the entire universe.
At the objective level, $tamas$ is inertia, $rajas$ is action or transformation and $sattva$ is the relative balance or equilibrium between $tamas$ and $rajas$. The interplay between the three sets up oscillations in the objective and the mental levels. In Yoga, the objective is to achieve the cessation of the fluctuations of the mind.
Consciousness or pure awareness is by definition not an object and therefore it does not have attributes. It must for the same reason be beyond the categories

of the living or dead. It must be beyond inertia, or change or fluctuations. It is extraordinary that in this analysis the qualities that are associated with objects become describable by an internal order.

The $guṇas$ do not admit of any further breakdown. This defines a position that is different from that of Aristotelian physics [14]. The three $guṇas$ are present in all objects and we can isolate one only in terms of the momentary strength of one in relation to the other in a process. Their fluctuations mark the universal "internal clock" of worldly processes.

In the Sāṃkhya, the effect is the cause in a new form, and this is why the system is also called $pariṇāmavāda$, or theory of transformation. Between the cause and effect is a relation of identity-and-difference, that is identity of stuff but difference of form ($bhedābheda$). The method at the basis of the Sāṃkhya and the *Nyāya Sūtra* may be seen in the *Yoga Sūtra* as well. In the *Yoga Sūtra* 3.13 three aspects of change are identified: transformation of a thing ($dharmi$) into a property ($dharma$), transformation of a property into a mark ($lakṣaṇa$), and the transformation of a mark into a condition ($avasthā$).

4. The form of the Nyāya syllogism

The five parts of the Nyāya syllogism spring from the idea of $bandhu$ that is fundamental to Vedic thought. The $bandhu$ is the equivalence between two different systems, which ordinarily are the microworld, the macroworld, and the individual's cognitive system [5].

The Nyāya syllogism first sets up the propositional system with its two components (two parts) and then identifies another well known system to which the first is supposed to have a $bandhu$-like relationship (third and fourth parts). The conclusion (fifth part) can be made only after the preliminaries have been formally defined.

The appeal to the $bandhu$ in the syllogism is to acknowledge the agency of the subject who can be, without such knowledge, open to invalid perception. One can see how in systems that do not accept transcendental reality (such as Aristotle's or Buddhist), a simplification from the five-part to the three-part syllogism would be most natural.

The Nyāya considers the following five elements essential to correct reason [12]:

1. The reason (evidence) must be present in the case under consideration;
2. It must be present in another case similar to the one under consideration;
3. It must not be present in cases dissimilar to the case under consideration;
4. It must be such that the proposition it tries to establish is not contradicted by another already established truth;
5. It must be such that thre should not be another evidence or reason establishing the opposite thesis, to counterbalance the thesis it tries to establish.

5. Vaiśeṣika and other views

This school of "individual characteristic" is supposed to have been founded by Kaṇāda, the son of Ulūka [15],16]. *Vaiśeṣika Sūtras* describe a system of physics and metaphysics. Its physics is an atomic theory of nature, where the atoms are distinct from the soul, of which they are the instruments. Each element has individual characteristics (*viśeṣas*), which distinguish it from the other non-atomic substances (*dravyas*): time, space, soul, and mind. The atoms are considered to be eternal. There are six fundamental categories (*padārtha*) associated with reality: substance (*dravya*), quality (*guṇa*), motion (*karman*), universal (*sāmānya*), particularity (*viśeṣa*), and inherence (*samavāya*). The first three of these have a real objective existence and the last three are products of intellectual discrimination. Each of these categories is further subdivided as follows.

There are nine classes of substances, some of which are nonatomic, some atomic, and others all-pervasive. The nonatomic ground is provided by the three substances ether (*ākāśa*), space (d*iś*), and time (*kāla*), which are unitary and indestructible; a further four, earth (*pṛthivā*), water (*āpas*), fire (*tejas*), and air (*vāyu*) are atomic composed of indivisible, and indestructible atoms (*aṇu, paramāṇu)*; self (*ātman*), which is the eighth, is omnipresent and eternal; and, lastly, the ninth, is the mind (*manas*), which is also eternal but of atomic dimensions, that is, infinitely small.

There are seventeen qualities (*guṇa*), listed in no particular order as color or form (*rūpa*), taste (*rasa*), smell (*gandha*), and touch (*sparśa*); number (*saṃkhyā*), size or dimension (*parimāṇa*), separateness (*pṛthaktva*), conjunction (*saṃyoga*),

and disjunction (*vibhāga*); remoteness (*paratva*) and nearness (*aparatva*); judgment (*buddhi*), pleasure (*sukha*), pain (*duḥkha*), desire (*icchā*), aversion (*dveṣa*), and effort (*prayatna*). These qualities are either physical or psychological. Remoteness and nearness are interpreted in two different ways: temporally or spatially. This list is not taken to be comprehensive because later sound is also described as a quality. But there is a fundamental difference between sound and light. Sound is carried by the non-atomic *ākāśa*, whereas light, implied by *rūpa*, is carried by tejas atoms. But even sound is sometimes seen as a specific characteristic of atoms. There are five different types of motion (*karman*) that are associated with material particles or the organs of the mind: ejection, falling (attraction), contraction, expansion, and composite motion.

Universals (*sāmānya*) are recurrent generic properties in substances, qualities, and motions. Particularities reside exclusively in the eternal, non-composite substances, that is, in the individual atoms, souls, and minds, and in the unitary substances ether, space, and time. Inherence (*samavāya*) is the relationship between entities that occur at the same time. This provides the binding that we see in the various categories so that we are able to synthesize our experience.

The Vaiśeṣika atomic structure characterizes four of the five Sāṃkhyan *mahābhūtas*; the fifth, ether, is non-atomic and all-pervasive. Some of the Vaiśeṣika *guṇas* correspond to the Sāṃkhyan *tanmātras*. In Sāṃkhya the *tanmātras* come first, in Vaiśeṣika atoms are primary.

In the medieval period, Dignāga (c. 500 CE) argued that inference is a function of three terms: the property to be inferred (*sādhya*), the inferential mark (*sādhana*), and the locus (*pakṣa*). Kumārila Bhaṭṭa (c. 700 CE) argued that language can generate cognition of non-existent entities in what are empty terms (such as “horned rabbit” or the “son of a barren woman”). Udayana (10th century) refuted the Buddhist view of the momentariness of all entities.

6. Navya Nyāya

In the thirteenth century, the Navya Nyāya (New Logic) system was founded by Gaṅgeśa Upādhyāya of Mithilā [17],[18],[19]. Its development was influenced by the work of earlier philosophers Vācaspati Miśra (10th century) and Udayana. It developed a highly technical language to formulate and solve problems in logic and epistemology.

Gaṅgeśa's book *Tattvacintāmaṇi* ("Thought-Jewel of Reality") dealt with important questions in logic, set theory, and epistemology, which improved the Nyāya scheme. It systematized the Nyaya categories of perception (*pratyakṣa*), inference (*anumāna*), analogy (*upamāna*), and testimony (*śabda*).
A property with an empty domain was taken to be fictitious or unreal and non-negatable. Negation was considered a valid operation only on real properties. This could be considered to generate a three-valued table. If P, N, and U represent "positive", "negative", and "unnegatable", then we have the truth table [12]:

w	not-w
P	N
N	P
U	U

Knowledge was taken to be analyzed into three kinds of epistemological entities in their interrelations: "qualifier" (*prakāra*); "qualificand", or that which must be qualified (*viśeṣya*); and "relatedness" (*saṃsarga*). For each of these was the corresponding abstract entity: qualifierness, qualificandness, and relatedness. The knowledge expressed by the judgment "This is a red flower" was then analyzed in the following sense: "The knowledge that has a qualificandness in what is denoted by *this* is conditioned by a qualifierness in *red* and conditioned by another qualifierness in *flowerness*."
Various relations were introduced, such as direct and indirect temporal relations, *paryāpti* relation (in which a property resides in sets rather than in individual members of those sets), *svarūpa* relation (which holds, for example, between an absence and its locus), and relation between the cognition of a knowledge and its object.
The concept of "limiterness" (*avacchedakatā*) was put to many different uses. If a field has fire in one region and not in another, the Navya Nyāya proposition would be expressed as: "The field, as limited by the region A, possesses fire, but as limited by the region $\neg A$ possesses the absence of fire." In the manner, limitations of time, property, and relation were also described.
The notion of negation was developed beyond specifying it with references to its limiting counterpositive (*pratiyogin*), limiting relation, and limiting locus.

Questions such as the following were asked: Is one to recognize, as a significant negation, the absence of a thing A so that the limiter of the counterpositive A is not A-ness but B-ness? Gaṅgeśa believed that the answer to these three questions was in the negative. He, however, believed that the absence of an absence itself could lead to a new property.

7. Concluding remarks

The important role that logic has enjoyed in the Indian cultural area is due to belief in the overarching Vedic system that knowledge is of two kinds: objective and subjective. All objective knowledge is governed by logic, whereas the knowledge related to the experiencing subject is extra-logical or transcendent. Since ordinary science must be objective, logic is essential in all philosophy and scientific disciplines.

The philosophical systems of the Nyāya and the Vaiśeṣika, together with the philosophical school related to grammar, kept up a continuing debate over centuries that led to the consideration of several subtle problems. The culmination of the Indian logical tradition was in the Navya Nyāya school.

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